On the phase shift of an electron wave due to a potential : a relativistic approach

JOUFFREY BERNARD
[1]LMSS-MAT, UMR ECP-CNRS 8579, Grande Voie des Vignes, 92290, Châtenay-Malabry

bernard.jouffrey@ecp.fr

1- INTRODUCTION

The phase shift of the wave associated to the electron plays a fundamental role in many explanations of contrasts, particularly in conventional electron microscopy. In holography, the calculation of the phase shift is essential. At voltages, as 200 keV, the question is to understand if the models we used, based on the Glauber model, is sufficient to give the correct phase shift. In this paper we intend to present a relativistic approach to this question.

1. EXPRESSION OF THE PHASE SHIFT

1.1 Projected potential

The well-known result for the phase shift is

$$\chi = -\frac{m}{\hbar^2 k}\int_{-\infty}^{z} V \partial z = -\frac{p}{\lambda E_c}\int_{-\infty}^{z} V \partial z$$

We consider here that the phase shift is given by a projected potential.
m is the mass of the electron, k the wave vector, V the potential energy related to the target. V is a function of coordinates x, y, z, but we integrate on z only. It is considered that there is no scattering, We have a phase shift only.

1.2 Relativistic approach

We arrive easily to the equations [(1)]

$\left(\Delta + k^2\right)\psi\left(x,y,z\right) - \frac{2m}{\hbar^2}\gamma V_r \psi\left(x,y,z\right) = 0$ where γ is the relativistic parameter. We can show easily that V_r is a retarded potential. Finally we arrive to a formula

$\chi = -\frac{1}{\hbar v}\int V_r[\gamma(r \pm \beta z)]dz$ taking into account the Lorentz transformation. The sign of the correction in βz depends on the geometry of the experiment which is considered.

2. APPLICATION

The relativistic model has been used to simulate the situation of reference (2).

2.1 Geometry

A latex sphere (Fig.1) is deposited on a thin substrate. It is positively charged under the beam.

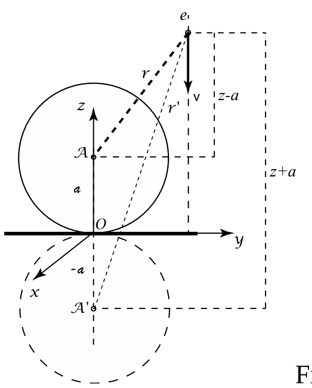

Figure 1

2.2Simulations

The first conditions, not so obvious, are to write the conditions to get the potential to be zero at z infinite (>0), and at the level of the substrate (z=0). We see on fig. 2 the difference for a latex sphere charged with 400 elementary positive charges.

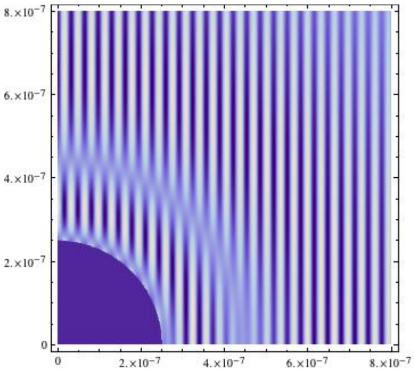

Fig. 2 a) 200kV and relativistic correction

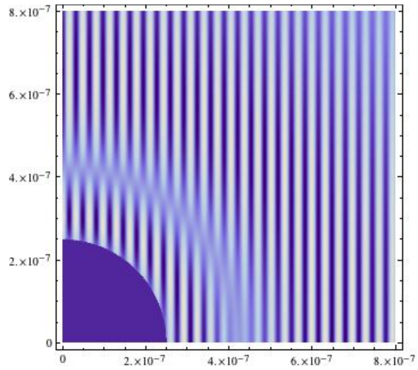

Fig. 2 b) 200 kV and no relativistic correction

Figure 1. Simulations on a quarter of a latex sphere

3. CONCLUSION

A relativistic approach of the phase shift of a wave under the influence of a scalar field has been carried out. It has been used to simulate the contrast of a charged latex sphere (positive charge) in an holographic experiment. We shall show, at the congress, the expression of this relativistic phase shift. This approach can be used in many other situations.

REFERENCES

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